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KAN GUIDE ON THE EVALUATION AND EXPRESSION OF UNCERTAINTY IN MEASUREMENT

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LIST OF AMANDMENT

No.	Date	Part number revised	Brief description of changes	Part revision number
1.	12/06/08	Identification document DPLP 13	Identification document changed to KAN-G-01	2
2.	07/01/11	Related terms	Change of term of measurement traceability (on VIM 93) to metrological traceability (on VIM 07), and based on ILAC Policy on change of term of Best Measurement Capability (BMC) to Calibration and Measurement Capability (CMC) on August 2009.	3
3.	07/04/16	Annex as reference	Add annex as reference	4

KAN GUIDE ON THE EVALUATION AND EXPRESSION OF UNCERTAINTY IN MEASUREMENT

1. INTRODUCTION

Compliance testing sometimes involves measured values, which lie close to the zone of uncertainty. A different method of uncertainty evaluation by foreign authority could mean rejection of a container of goods destined for import because of expansion of the recalculated zone of uncertainty.

In the era of global marketplace it is imperative that the method for evaluating and expressing uncertainty be uniform throughout the world so that measurements performed in different countries can be easily compared. The internationally accepted guidance for the evaluation of measurement uncertainty is the *ISO "Guide to the Expression of Uncertainty in Measurement".*

This document describes the principles on the evaluation of measurement uncertainty for calibration and testing laboratories to meet the requirement of ISO/IEC 17025 on *'General Requirements for the Competence of Calibration and Testing Laboratories'.*

The method of evaluating measurement uncertainty described in this document is in accordance with ISO *"Guide to the Expression of Uncertainty in Measurement".*

This document gives the recommended method for evaluating measurement uncertainty that is applicable for calibration and testing laboratories which wants to be accredited by National Accreditation Body of Indonesia (KAN) based on ISO/IEC 17025.

Regarding one of the important factors in the accreditation of calibration laboratories, that is *Best Measurement Capability (CMC)*, this document also gives general guidance in evaluating CMC.
To assist laboratories in implementing the method in this document worked examples on the evaluation of measurement uncertainty for calibration and testing laboratories and the evaluation of Best Measurement capability will be given in Supplements.

2. TERMS AND DEFINITIONS

The following terms and definitions are given to assist the users in understanding this document.
Cross-references to the ISO/IEC Guide 99 *Vocabulary of Basic & General Terms of Metrology* are respectively given in the square brackets.

Quantity [ISO/IEC Guide 99 1.1]
Property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference

Value (of a quantity) [ISO/IEC Guide 99 1.19
Number and reference together expressing magnitude of a quantity

True value (of a quantity) [ISO/IEC Guide 99 2.11]
Quantity value consistent with the definition of a quantity

NOTE 1 In the Error Approach to describing measurement, a true quantity is considered unique and, in recognize that, owing to the inherently incomplete amount of detail in the definition of a quantity, there is not a single true quantity value but rather a set of true quantity values consistent with the definition. However, this set of values is, in principle and in practice, unknown able. Other approaches dispense altogether with the concept of true quantity value and rely on the concept of metrological compatibility of measurement results for assessing their validity.
NOTE 2 In the special case of a fundamental constant, the quantity is considered to have a single true quantity value.

NOTE 3 When the definitional uncertainty associated with the measurand is considered to be negligible compared to the other components of the measurement uncertainty, the measurand may be considered to have an "essentially unique" true quantity value. This is the approach taken by the GUM and associated documents, where the word "true" is considered to be redundant

Conventional value (of a quantity) [ISO/IEC Guide 99 2.12]
Quantity value attributed by agreement to a quantity for a given purpose.

Measurement [ISO/IEC Guide 99 2.1]
Process of experimentally obtaining one or more quantity values that can be reasonably be attributed to a quantity

Measurand [ISO/IEC Guide 99 2.3]
Quantity intended to be measured

Result of a measurement [ISO/IEC Guide 99 2.9]
Set of quantity values being attributed to a measurand together with any other available relevant information

NOTE 1 A measurement result generally contains "relevant information" about the set of quantity values, such that some may be more representative of the measurand than others.This may be expressed in the form of a probability density function (PDF)

NOTE 2 A measurement result is generally expressed as a single measured quantity value and a measurement uncertainty. If the measurement uncertainty is considered to bel negligible for some purpose, the measurement result may be expressed as a single measured quantity value. In many fields, this is the common way of expressing a measurement result.

NOTE 3 In the traditional literature and in the previous edition of the VIM, measurement result was defined as a value attributed to a measurand and explained to mean an indication, or an uncorrected result, or a corrected result, according to the context.

Correction [ISO/IEC Guide 99 2.53]
Compensation for an estimated systematic effect
Accuracy (of a result of a measurement) [ISO/IEC Guide 99 2.13]
Closeness of agreement between a measurand quantity value, and the true quantity value of the measurand.

NOTE 1 The concept 'measurement accuracy' is not a quantity and is not given a numerical quantity value. A measurement is said to be more accurate when it offers a smaller measurement error.
NOTE 2 The term "measurement accuracy" should not be used for measurement trueness and the term measurement precision should not be used for "measurement accuracy", which , however, is related to both these concepts.
NOTE 3 "Measurement accuracy" is sometimes understood as closeness of agreement between measured quantity values that are being attributed to the measurand.

Repeatability condition of measurement [ISO/IEC Guide 99 2.20]
Condition of measurement, out of a set of conditions that includes the same measurement procedure, same operators, same measuring system, same operating conditions and same location, and replicate measurements of the same or similar objects over a short period of time

Repeatability [ISO/IEC Guide 99 2.21]
Measurement precision under a set of repeatability conditions of measurement

Reproducibility condition of measurement [ISO/IEC Guide 99 2.24]
Condition of measurement, out of a set of conditions that includes different locations, operators, measuring systems, and replicate measurements on the same or similar objects

Reproducibility [ISO/IEC Guide 99 2.25]
Measurement precision under a set of reproducibility conditions of measurement

Error (of a measurement) [ISO/IEC Guide 99 2.16]
Measured quantity value minus a reference quantity value

NOTE 1 The concept of "measurement error" can be used both :

- *a) when there is a single reference quantity value to refer to, which occurs if a calibration is made by means of a measurement standard with a measurement quantity value having a negligible measurement uncertainty or if a conventional quantity value is given, in which case the measurement error is known, and*

b) *if a measurand is supposed to be represented by a unique true quantity value or a set of true quantity values of negligible range, in which case the measurement error is not known.*

NOTE 2 Measurement error should not be confused with production error to mistake

Random error (of a measurement) [ISO/IEC Guide 99 2.19]
Component of measurement error that in replicate measurements varies in an unpredictable manner

NOTE 1 A reference quantity value for a random measurement error is the average that would ensue from an infinite number of replicate measurements of the same measurand
NOTE 2 Random measurement errors of a set of replicate measurements form a distribution that can be summarized by its expectation, which is generally assumed to be zero, and its variance
NOTE 3 Random measurement error equals measurement error minus systematic measurement error.

Systematic error [ISO/IEC Guide 99 2.17]
Component of measurement error that in replicate measurements remains constant or varies in a predictable manner

NOTE 1 A reference quantity value for a systematic measurement error is a true quantity value, or a measured quantity value of a measurement standard of negligible measurement of uncertainty or a conventional quantity value.
NOTE 2 Systematic measurement error, and its causes, can be known or unknown. A correction can be applied to compensate for a known systematic measurement error.
NOTE 3 Systematic measurement error equals measurement error minus random measurement error.

Correction [ISO/IEC Guide 99 2.53]
Compensation for an estimated systematic effect
NOTE: The compensation can take different forms, such as an added or a factor, or can be deduced from a table.

Uncertainty [ISO/IEC Guide 99 2.26]
Non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used.

Standard uncertainty [ISO/IEC Guide 99 2.30]
Measurement uncertainty expressed as a standard deviation

Type A evaluation (of uncertainty) [ISO/IEC Guide 99 2.28]
Evaluation of a component of measurement uncertainty by statistical analysis of measured quantity values obtained under defined measurement conditions

Type B evaluation (of uncertainty) [ISO/IEC Guide 99 2.29]
Evaluation of a component of measurement uncertainty determined by means other than a Type A evaluation of measurement uncertainty

Combined standard uncertainty [ISO/IEC Guide 99 2.31]
Standard measurement uncertainty that is obtained using the individual standard measurement uncertainties associated with the input quantities in a measurement model

Coverage factor [ISO/IEC Guide 99 2.38]
Number larger than one by which a combined standard measurement uncertainty is multiplied to obtain an expanded measurement uncertainty.

Expanded uncertainty [ISO/IEC Guide 99 2.35]
Product of a combined standard measurement uncertainty and a factor larger than the number one.

3. GENERAL CONCEPTS

The objective of a measurement is to determine the value of the measurand that involve specification of the measurand, the method of measurement and the procedure of measurement.

In general, the result of a measurement is only an estimate or approximation of the value of the measurand, therefore the result is complete only when accompanied by the statement of the uncertainty of the estimate.

Uncertainty is a measure of the dispersion that may reasonably be associated with the measured value. It gives a range, centered on the measured value, within which, to a stated probability, the true value lies.

The uncertainty of the result of a measurement reflects the lack of exact knowledge of the value of the measurand. The result of a measurement after correction for recognized systematic effects is still only an estimate of the value of the measurand because of the uncertainty arising from random effects and from imperfect correction of the systematic effects.

The concept of uncertainty is based on the observable quantities obtained by measurement; this differs from the ideal concept of error based on the unknowable quantities. Traditionally, an error of a measurement result is considered as having two components, namely random component and systematic component. Random error presumably arises from unpredictable or stochastic temporal and spatial variations of influence quantities. Systematic error arises from a recognized effect of an influence quantity of a measurement result.

The difference between error and uncertainty should always be borne in mind. For example, the result of a measurement after correction can unknowably be very close to the unknown value of the measurand, and thus have negligible error, even though it may have a large uncertainty.

4. SOURCES OF UNCERTAINTY

In practice there are many possible sources of measurement uncertainty, including:

- Incomplete definition of the measurand
- Imperfect realization of the definition of the measurand
- Sampling - the sample measured may not represent the defined measurand
- Inadequate knowledge of the effects of environmental conditions on the measurement process or imperfect measurement of environmental conditions
- Personal bias in reading analogue instruments
- Instrument resolution or discrimination threshold
- Values assigned to measurement standards and reference materials
- Values of constants and other parameters obtained from external sources and used in the data reduction algorithm
- Approximation and assumptions incorporated in the measurement method and procedure
- Variations in repeated observations of the measurand under apparently identical conditions
- In addition to those general sources of uncertainty, the specific sources of uncertainty in testing may include, but not limited to:
- Non-representative sampling
- Non-homogeneity nature of the sample
- Contamination during sampling and sample preparation
- Purity of reagents and solvents
- Matrix effects and interference
- Blank corrections

5. CLASSIFICATION OF COMPONENTS OF UNCERTAINTY

Generally, the uncertainty of a measurement consists of several components which may be classified into two categories in accordance with the method used to estimate their numerical values:

- Type A : those which are evaluated by statistical analysis of series of observations
- Type B : those which are evaluated by other means other than statistical analysis of series of observations

Classification of uncertainty components into type A and type B does not always have simple correspondence with the commonly used classification of uncertainty components as “random” and “systematic”. The nature of an uncertainty component is conditioned by the use made of the corresponding quantity, that is, on how that quantity appears in the mathematical model that describes the measurement process. When the corresponding quantity is used in a different way, a “random” component may become a “systematic” component and vice versa. Thus the terms “random uncertainty” and “systematic uncertainty” can be misleading when generally applied. An alternative nomenclature that might be used is:

- “Uncertainty component arising from a random effect,”
- “Uncertainty component arising from a systematic effect.”

Random effect is one that gives rise to a possible random error in the current measurement process and a systematic effect is one that gives rise to possible systematic error in the current measurement process.

In practical measurement, an uncertainty component arising from systematic effect may in some cases be evaluated by type A evaluation while in other cases by type B evaluation, as may be an uncertainty component arising from a random effect.

6. MODELING THE MEASUREMENT

In relation with the evaluation of measurement uncertainty, measurement models need the clear statement of measured quantities, and the quantitative expression shows the relation between the value of measurand and independence parameters where the measurand depends on. Those parameters may be other measurand, quantities those are not measured directly or a constant. The function, which relates the measurand and input quantities is called as measurement model.

In most of measurement processes a measurand Y is determined from N other quantities i.e. $X_1, X_2, \ldots, X_N$ through a functional relationship:
$Y = f(X_1, X_2, \ldots, X_N)$

The input quantities $X_1, X_2, \ldots, X_N$ *upon* which the measurand Y may be viewed as other measurands and may themselves depend other quantities, including corrections and correction factors for recognized systematic effects, thereby leading to a complicated functional relationship *f* that may never be written down explicitly.

The input quantities $X_1, X_2, \ldots, X_N$ may have values and uncertainties those are directly determined in the current measurement process (such as from a single observation, repeated observation, determination of correction to instruments reading and correction from influence quantities) or obtained from external sources (such as quantities associated with calibrated measurement standards, certified reference materials, and reference data from handbook).

An estimate of the measurand Y, denoted by y, obtained from equation (1) using the estimates of input quantities $X_1, X_2, \ldots, X_N$, for the values of the N quantities $X_1, X_2, \ldots, X_N$, therefore the estimate of measurand y, which is the result or the measurement process, is given by:
$y = f(X_1, X_2, \ldots, X_N)$

Where it is assumed that each input estimate has been corrected for all recognized systematic effect that is significant for the output estimate.

The estimated standard deviation associated with output estimate, termed as combined standard uncertainty (denoted as $u_c(y)$) is obtained by appropriately combining the estimated standard deviation of each input estimate x_i that is termed as standard uncertainty (denoted as $u(x_i)$)

Each standard uncertainty $u(x_i)$ is obtained either from type A or type B evaluation.

7. IDENTIFICATION OF UNCERTAINTY SOURCES

When the measurement process has been expressed in the mathematical model, the uncertainty sources related to the measurement processes shall be well identified to avoid the overestimate or underestimate values of uncertainty.

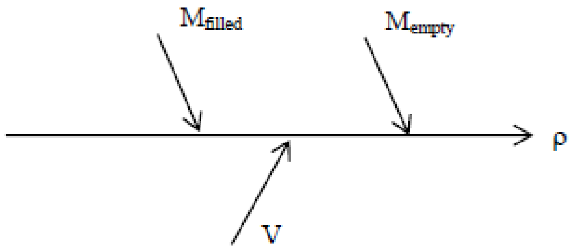
To help the identification process, especially for the measurements those involve many input and influence quantities, the use of cause and effect diagram may be able to simplify the processes.

The following procedure can be used as the guidance to make cause and effect diagram:

1. Write down the complete equation represent the measurement processes based on the results of measurement modeling. The parameters shown in the equation build the major branch of the diagram.

   Example:
   The measurement of liquid density based on weighing method: Mathematical model: $\rho = (m_{isi} - m_{kosong})/V$

   Where:
   ρ is the density of liquid
   m_{isi} *is the mass of (volumetric flask + liquid) obtained from the balance reading*
   m_{kosong} *is the mass of volumetric flask based on the balance reading*
   V is the volume of volumetric flask

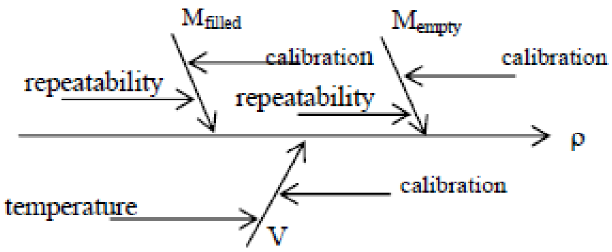


2. Look at each step in the methods and add another factors into the diagram, which form branch of the major branch of the diagram.

   In the liquid density measurement process, the calibrated balance and calibrated volumetric flask are used. Measurement is repeated n-times

   In this process the following uncertainty contribution must be considered:
   - balance calibration
   - repeatability of weighing
   - calibration of volumetric flask
   - repeatability of volume measurement
   - effect of temperature to the capacity of volumetric flask

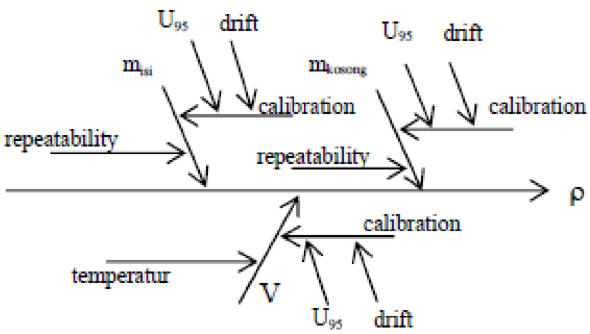
By adding those above factors in the diagram, we get:



3. For each branches, add another factors those give contribution until all significant factors included in the diagram.

   Based on the uncertainty sources identified in point (2), then we must consider the following:

   - the calibration certificate of balance: Expanded uncertainty contained in the certificate Drift of the balance indication based on the historical data
   - the calibration certificate of volumetric flask Expanded uncertainty contained in the certificate Drift of the volumetric flask based on the historical data
   - measurement of ambient temperature Expanded uncertainty contained in the calibration certificate of thermometer Distribution of ambient temperature based on the monitoring results



When the identification process has been finished, the next step is classifying the uncertainty components to determine the evaluation methods.

8. TYPE A EVALUATION OF STANDARD UNCERTAINTY

When measurement is repeated several times, the mean value and the standard deviation can be calculated. The standard deviation describes the dispersion of applicable to the whole population of possible measured values.

In most cases, the best available estimate of the expectation or expected value of a quantity that varies randomly, and for which n independent repeated observations have been obtained under the same conditions of measurement, is the arithmetic mean or average of the n observations

$$\bar{x} = \frac{1}{n}\sum_{1}^{n} x_i$$

The standard deviation is an estimate of the dispersion of the population from which the n values are taken

$$s(x_i) = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

After taking one set of *n* repeated measurements we were to take a second set of *n* measurements and we could again calculate the mean. It most likely would be slightly different from the first mean. The estimate of the dispersion of the population mean can be calculated as the experimental standard deviation of the mean (ESDM)

$$s(\bar{x}) = \frac{s(x_i)}{\sqrt{n}}$$

The type A standard uncertainty u (xi) for a quantity determined from n independent repeated observations is the ESDM:

$$u(x_i) = s(\bar{x})$$

Sometimes it is necessary to know the number of degrees of freedom, for a set of n measurements for which we obtain a mean, the degrees of freedom is:

$$v_i = n - 1$$

For a well-characterized measurement under statistical control, a pooled experimental standard deviation SP, with degrees of freedom v_p based on M series of observations of the same variable may be available. The pooled experimental standard deviation is determined be:

$$s_p = \sqrt{\frac{\sum_{i=1}^{M} v_i s_i}{\sum_{i=1}^{M} v_i}}$$

$$v_p = \sum_{i=1}^{M} v_i$$

Where s_i is the experimental standard deviation from one series of mi independent repeated observations, and has degrees of freedom:

$v_i = m_i - 1$

If the measurement result *x* of the same variable is determined from n independent observations, the type A standard uncertainty u can be estimated by:

$$u(x_i) = \frac{s_p}{\sqrt{n}}$$

There are many methods of determining type A standard uncertainty, the most common calculation is the ESDM, the next most common type evaluation is determination of standard uncertainties from *fitted curves*.

For example suppose we wish to fit a straight line to some data, the straight line is described by the equation:

$y = a + bx$

The difference between an actual data point and the corresponding value calculated from the equation for the curve is called residual. In a curve fitting process the intention is to find values of *a* and *b* such that the sum of the squares of residuals (SSR) is minimized.

$$SSR = \sum (y_i - a - bx_i)^2$$

The scatter of the data points around the fitted curve can be described by an estimate of standard deviation, often called as the standard error of the y values calculated using the curve, which is calculated by:

$$s = \sqrt{\frac{SSR}{v}}$$

Where v is the number of the degrees of freedom, which can be calculated by:

v = *number of data points – number of coefficients fitted*

v = *number of data points – 2 for a straight line*

As with the mean of repeated observations, for the curve, associated standard uncertainty is obtained from the estimate of standard deviation.

$u = s$

The curve fitting process is not limited to a straight line, generally the fitted curve can be expressed as:

$y = f(x)$

Although the calculation of coefficients of the fitted curve and evaluating its uncertainty is seem difficult, many of commercial software packages have built in function for the curve fitting (regression) calculation.

9. TYPE B EVALUATION OF STANDARD UNCERTAINTY

Type B evaluation of standard uncertainty is obtained by means other than the statistical analysis of a series of observations that usually based on scientific judgment using all relevant information available, which may include:

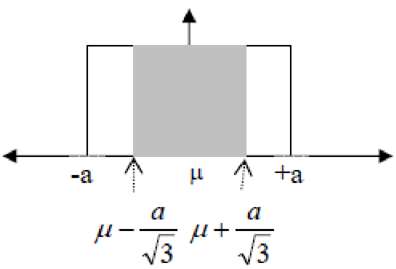
- Previous measurement data
- Experience with, or general knowledge of the behavior and property of relevant materials and instruments
- Manufacturer's specification
- Data provided in calibration and other reports
- Uncertainties assigned to reference data taken from data book

The simplest example of type B evaluation is the use of uncertainty reported in the certificate of standard. To obtain the standard uncertainty, the expanded uncertainty on the certificate is divided by coverage factor given on the certificate. In the absence of a value for the coverage factor, a factor of 2 may be used if the expanded uncertainty has a 95% confidence level.

In other case the uncertainty is given as the specified limits, $\pm$ a, the probability distribution can be estimated from the available information, which may take one of the following distributions:

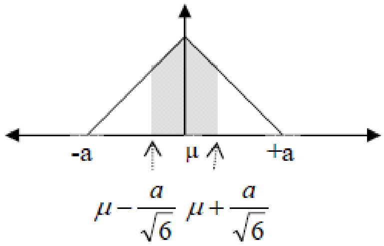
Rectangular Probability Distribution

It is used if limits can be determined, but the value of the measurand is just likely to be anywhere in the range. The standard uncertainty is obtained by dividing the semi-range *'a'*



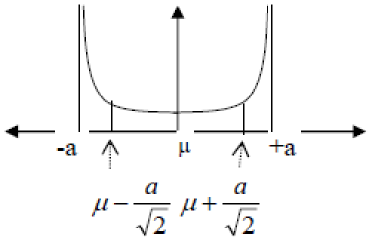
Triangular Probability

Distribution It is used when there is evidence that the values near the mean are the most probable value, as the limits decreased, the probabilities decreases to zero. The standard uncertainty is



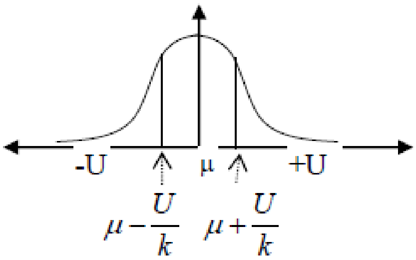
U-Shape Probability Distribution

This distribution occurs in several area of metrology. An example is the distribution for uncertainties arising from the radio frequency connector reflections. It may also be applicable to air temperature variations where the temperature control produce regular temperature excursion between limits. The standard uncertainty is obtained by dividing



Gaussian or Normal Distribution

This distribution form can be assumed for an uncertainty that defines a confidence interval having given level of confidence of say 95% or 99%. The standard uncertainty is obtained by dividing quoted uncertainty by the appropriate coverage factor based on t-distribution table, i.e. u = U / k; where U is the expanded uncertainty for specified confidence level and k is the coverage factor.



For type B evaluation of standard uncertainty, rectangular distribution is a reasonable default model in the absence of any other information. But if it is known that values of the quantity in question near the center of the limits, a triangular or normal distribution may be a better model.

Type B standard uncertainty is obtained from a priori probability distributions. It is simplicity assumed that the probability distribution is exactly known. In most cases, we can assume that the degrees of freedom for such standard uncertainty as infinite. This is reasonable assumption as it is a common in practice to choose a type B uncertainty that the probability of the concerned quantity lying outside the uncertainty band is extremely small.

10. SENSITIVITY COEFFICIENTS

The sensitivity coefficient is one of the aspects in evaluating measurement uncertainty that causes difficulty. The sensitivity coefficients convert all uncertainty components to the same unit as the measurand. This is necessary precondition to combining uncertainty components having different units.

The sensitivity coefficients also give a scaling of weighing function for each uncertainty component; those describe how the output estimate varies with the changes in the value of the input estimates

Evaluations of the sensitivity coefficients can be done based on the partial differentiation of a function represent the mathematical model of a measurement.

$$c_i = \partial f / \partial x_i$$

The sensitivity coefficients sometimes determined experimentally, by varying specified input quantity while holding the remaining input quantities constant.

Sensitivity coefficients sometimes can be determined experimentally by varying specified input quantities and keeps constant another input quantity.

If $y = f(x_1, x_2, x_3,...)$ and uncertainty of each input quantity expressed as $u(x_i)$, contribution of an input quantity $u_i(y)$ to the uncertainty of the measurand $u_c(y)$can also be obtained by using the following equation:

$$u_1(y) = c_1 u(x_1) = f(x_1 + u(x_1), x_2, x_3, ...) - f(x_1, x_2, x_3,...)$$
$$u_2(y) = c_2 u(x_2) = f(x_1, x_2 + u(x_2), x_3, ...) - f(x_1, x_2, x_3,...)$$

Etc.

At this time much software has built in mathematical function, this makes calculation of uncertainty contribution using the above equation can be easier than evaluate the partial differentiation of the measurand for each input quantities.

11. COMBINED STANDARD UNCERTAINTY

The combined standard uncertainty of a measurement, denoted by $u_c(y)$, is taken to represent the estimated standard deviation of the result. It is obtained by combining the individual standard uncertainties of input estimate based on a first order Taylor series approximation of the measurement model. The method for combining standard uncertainty is often called the law of propagation of uncertainty.

For uncorrelated input quantities, the combined standard uncertainty of input estimate y can be written as

$$u_c(y) = \sqrt{\sum_{i=1}^{N}[c_i u(x_i)]^2} = \sqrt{\sum_{i=1}^{N}[u_i(y)]}$$

Where: $c_i = \partial f / \partial x_i$ and $c_i u(x_i) = u_i(y)$

In measurement processes, there some occasions where two or more input quantities are interdependent. The appropriate expression for the combined standard uncertainty associated with the result of measurement is:

$$u_c(y) = \sqrt{\sum_{i=1}^{N}[c_i u(x_i)]^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} c_i c_j u(x_i) u(x_j) r(x_i, x_j)}$$

The interdependence of two variables is characterized by their correlation coefficients, which
can be expressed as:

$$r(x_i, x_j) = \frac{u(x_i, x_j)}{u(x_i)u(x_j)}$$

Correlation can occur if the same measurement is used more than once in the same measurement process, however, its effect of the combined uncertainty may be positive, i.e. the uncertainty is increased of negative, which will lead to a reduction in the uncertainty

If a positive correlation is suspected but the correlation coefficient cannot be calculated simply, it is reasonable to assume a correlation coefficient of +1. If all of the input estimates are correlated with correlation coefficients of +1, the combined standard uncertainty of output estimate can be expressed as:

$$u_c(y) = \sqrt{\left[\sum_{i=1}^{N} c_i u(x_i)\right]^2}$$

For practical purpose in testing area, the following simple's rules for combining standard uncertainty are given:

- If models involving only a sum or difference of quantities,
  e.g. $y = (p + q + r + \ldots)$

$$u_c(y) = \sqrt{u(p)^2 + u(q)^2 + u(r)^2 + \ldots}$$

- If models involving only a product or quotient, e.g. $y = p.q.r...$ or $y = p/(q.r...)$

$$u_c(y) = y\sqrt{(u(p)/p)^2 + (u(q)/q)^2 + (u(r)/r)^2 + ...}$$

- If models involving only n-order function

e.g. $y = a^n$

$u_c(y) = ny\, u(a) / a$

12. EFFECTIVE DEGREES OF FREEDOM

To need of the calculation of the effective degrees of freedom associated with an uncertainty component is to allow correct selection of the value Student's t, and also gives an indication on the reliability of the uncertainty estimation.

A high number of degrees of freedom represent the large number of measurement, low dispersion, and high confidence of the value, in other hand, a low number of degrees of freedom correspond to a large dispersion or poorer confidence in the value.

Every component of uncertainty has an appropriate number of the degrees of freedom, v, assigned to it. For the mean value of n measurement the degrees of freedom is
v = *n-1*

For the value associate with a fitted curve or regression, the number of degrees of freedom is:
v = *number of data points – number of coefficients fitted*

For the uncertainty components estimate based on the knowledge of limits $\pm$ a, the ISO GUM gives a formula that is applicable to all distributions, that is:

$$\nu \approx \frac{1}{2}\left[\frac{\Delta u(x_i)}{u(x_i)}\right]^{-2}$$

Where:

$\frac{\Delta u(x_i)}{u(x_i)}$ Is the relative uncertainty of estimated limits

If all the uncertainty components have been combined, the number of degrees of freedom of the combined standard uncertainty need to be estimated, that is the effective degrees of freedom for the combined standard uncertainty which can be calculated using Welch-Satterthwaite formula:

$$\nu_{eff} = \frac{u_c^4}{\sum_1^n \frac{u_i^4(y)}{\nu_i}}$$

where:

v_{eff} is the effective number of degrees of freedom for combined standard uncertainty
v_i is the number of degrees of freedom of the i-the uncertainty components
$u_{i(y)}$ is the product $c_{iu}(x_i)$

Based on the effective number of degrees of freedom of the combined standard uncertainty, the coverage factor needed in obtaining expanded uncertainty for desired confidence level can be obtained from the t-distribution table, for 95% confidence level, it may be calculated by the formula:

$$k = 1.95996 + 2.37356/v + 2.818745/v^2 + 2.546662/v^3 + 1.761829/v^4 + 0.245458/v^5 + 1.000764/v^6$$

13. EXPANDED UNCERTAINTY

In order to have an adequate probability that the value of the measurand lies within the range given by the uncertainty.

The measure of uncertainty intended to meet adequate probability is termed as expanded uncertainty, denoted by symbol *U*, and is obtained by multiplying *uc(y)* by a coverage factor, denoted by symbol k:

$U = k \times u_c(y)$

International practice is to give a level of confidence of approximately 95% (95.45%). For the specified level of confidence, the k value varies with effective degrees of freedom.

In many cases, k equal to 2 can be used where the effective degrees of freedom is reasonably large, that is greater or equal to 30. If the effective degrees of freedom are relatively small, the value of *k* can be obtained from the t-distribution table.

14. REPORTING UNCERTAINTY

In practice, the amount of information necessary given in the testing and calibration report or certificate depends on its intended use

In reporting measurement result, the following information should be provided:

- Result of measurement
- Expanded uncertainty with coverage factor and level of confidence specified
- Description of measurement method used to calculate the results and its uncertainty
- Values and sources of all corrections and constants used in both the calculation and the uncertainty analysis
- Functional relationship $Y=f(X_1, X_2, \ldots)$ and any such sensitivity coefficients determined experimentally should be given.
- In reporting calibration or test results and their uncertainies, the following should be considered:
- The numerical value of measurement uncertainty should be given at most two significant figures.
- During the stage of the estimation and combination of uncertainty components, at least one more figure should be used to minimize rounding errors.
- If the rounding brings the numerical value of measurement uncertainty down by more than 5 %, the rounding up value should be used.
- The numerical value of the measurement result should in the final statement normally be rounded to the least significant figure in the value of the expanded uncertainty assigned to the measurement result.

15. STATEMENT OF COMPLIANCE WITH SPECIFICATION

Clause 5.10.3.1 of ISO/IEC 17025 on test report state: *"... if necessary, for the interpretation of test report, include: ... b) when relevant, the statement of compliance/non-compliance with specification..."*.

For the calibration report, clause of ISO/IEC 17025 states: *"... if statement of compliance was made, uncertainty of measurement shall be taken into account"*

In harmony with those clauses of ISO/IEC 17025, when a test and/or calibration is carried out to a stated specification and the client or the specification requires the statement of compliance, the reports must contain a statement indicating whether the test and/or calibration results show compliance with the specification.

Where the measurement uncertainty is relevant to the validity or application of the test and/or calibration results, or when a client's instruction requires so, or shen the uncertainty affects compliance to a specification limits, the expanded uncertainty of measurement shall be taken into account. In addition level of confidence and coverage factor for the uncertainty shall be reported.

When a specification describes an interval with an upper and lower limit, the ratio of the uncertainty of measurement to the specified interval should be reasonably small. For an uncertainty of measurement U and a specified interval *2T (2T=upper limit-lower limit),* the ratio
U:T is a measure of the test or calibration method in distinguishing compliance from noncompliance.

The simplest case is where the specification clearly states that the test and/or calibration result, extended by the uncertainty at a given confidence level shall not fall outside or within a defined specification limits or limits.

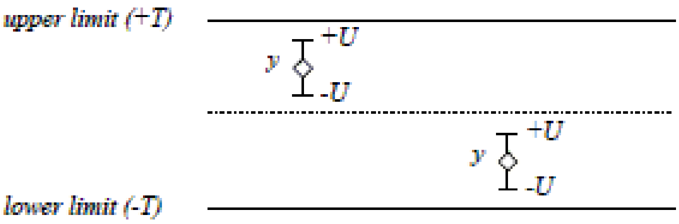
More often, the specification requires a statement of compliance in the certificate of report but makes no reference to taking into account the effect of uncertainty on the assessment of compliance. In such cases it may be appropriate for the user to make judgment of compliance, based on whether the test and/or calibration result is within the specified limits with no account taken of the uncertainty.

<u>Illustration:</u> the measured result for the diameter of a rod is 0.50 mm while the specification limit is between 0.45 mm to 0.55 mm, the user may conclude that the rod meets the requirement without considering the uncertainty of measurement.

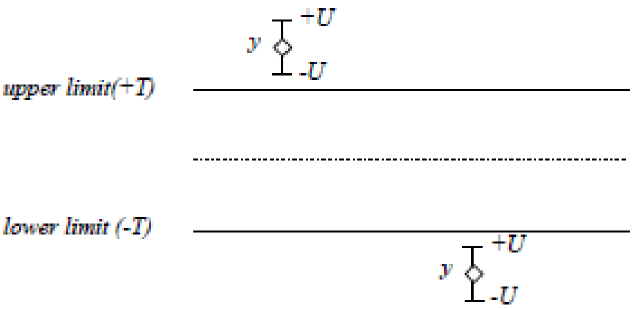
This often referred to as shared risk since the end user takes some of the risk that the product may not meet the specification after being tested with an agreed measurement method. In this case there is an implicit assumption that the uncertainty of the agreed measurement method is acceptable and it is important that it can be evaluated when necessary. National regulations can overrule the shared risk principle and can put the uncertainty risk on one party.

An agreements between the client and the laboratory, or code of practice or a specification may state that the accuracy or the adopted method is adequate and the uncertainty does not to be considered explicitly when judging compliance, similar considerations as for shared risk (above) apply in such circumstances.

In the absence of any criteria, test and/or calibration specifications, client's requirements, agreements, or code of practice, the following approach may be taken:
If the specification limits are not breached by the test and/or calibration result, extended by half of expanded uncertainty interval at a level of confidence of 95%, then compliance with the specification can be stated (as illustrated in the following figure).



When an upper specification limit is exceeded by the test result, even when it is extended downwards by half of the expanded uncertainty interval; or if a lower specification limit is breached, even when the test result is extended upwards by half of the expanded uncertainty interval, then *non-compliance* with the specification can be stated (as illustrated in the following figure):

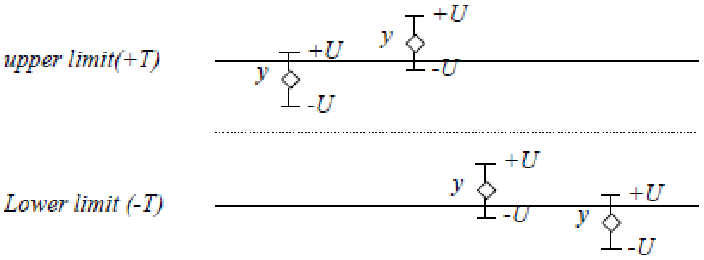


When the measured single value, without possibility of testing more samples from the same unit of product, falls sufficiently close to a specification limit such that half of the expanded uncertainty interval overlap the limit, *it is not possible to conform compliance or noncompliance at the stated level of confidence*. The test result and expanded uncertainty should be reported together with a statement indicating that neither compliance nor non-compliance was demonstrated. A suitable statement to cover these situation (as illustrated in the following figure), would be, for example

> *"the test result is above (below) the specification limit by a margin less than the measurement uncertainty; it is therefore not possible to state compliance / noncompliance based on a 95% level of confidence. However, where a confidence level of less than 95% is acceptable, a compliance / non-compliance statement may be possible"*

If the law requires a decision concerning rejection or approval, w*hen the measurement or testing results, lie within the specification range*, a statement of *compliance* could be made with a lower calculated and reported confidence level.

In other case, when *the measurement and testing result, lie outside the specification range*, a statement of *non-compliance* could be made with a lower calculated and reported confidence level



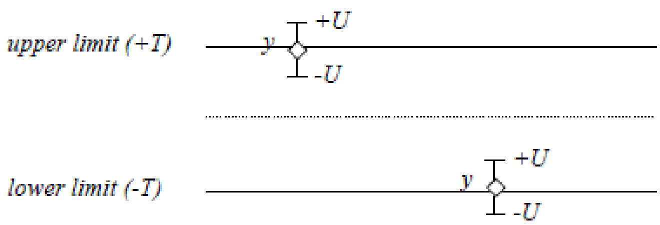
If the test result is exactly on the specification limit (as illustrated in the following figure), it is not possible to state compliance or non-compliance at the stated level of confidence, The measurement and/or test result should be reported together with statement indicating that neither compliance or non-compliance was demonstrated at the stated level of confidence. A suitable statement to cover these situation would be for example:

> “*The result is equal to the specification limit; it is therefore not possible to state either compliance or non-compliance at any level of confidence”*

If the law requires a statement concerning the assessment in the form of compliance or noncompliance, regardless of the level of confidence, the statement may be made depends on the definition of the specification:

> *If the specification limit is defined as “≤” or “≥”, and the test result is equal to specification limit, then compliance can be stated*

> *If the specification limit is defined as “<” or “>”, and the measurement and/or test result is equal to specification limit, then non-compliance can be stated.*



16. SUMMARY OF EVALUATION PROCEDURE

The following is guide to use these documents in practice:

- Derive or estimate the mathematical model of measurement process
- Determine the estimated value of input quantity,
- List all sources of uncertainty in the form of an uncertainty analysis
- Evaluate the type A standard uncertainty for repeatedly measured quantities
- Estimate the type B standard uncertainty based on the available information
- Evaluate the sensitivity coefficients for each input quantities
- Calculate the combined standard uncertainty
- Evaluate the effective degrees of freedom
- Calculate the expanded uncertainty of measurement result
- Report the result of the measurement and the associate expanded uncertainty and the coverage factor in calibration/testing report/certificate.
- If the statement of compliance with specification is necessary, evaluate compliance with specification based on the requirement of the standard and/or clients.

17. EVALUATION OF CALIBRATION AND MEASUREMENT CAPABILITY

CMC is a calibration and measurement capability available to customers under normal conditions:

- as published in the BIPM key comparison database (KCDB) of the CIPM MRA or,
- as described in the laboratory’s scope of accreditation granted by a signatory to the ILAC Arrangement.

The meaning of the terms Calibration and Measurement Capability , CMC, (as used in the CIPM MRA), and Best Measurement Capability, BMC, (as used historically in connection with the uncertainties stated in the scope of an accredited laboratory) are identical and should be interpreted similarly and consistently. BMC is defined as, “*the smallest uncertainty of measurement that a laboratory can achieve within scope of accreditation, when performing more or less routine calibrations of nearly ideal measurement standards intended to define, realize, conserve of reproduce a unit of that quantity of one or more of its values, or when performing more or less routine calibration of nearly ideal measuring instruments designed for the measurement of that quantity.”*

Under a CMC, the measurement or calibration should be :

- performed according to a documented procedure and have an established uncertainty budget under the management system
- performed on a regular basis (including on demand or scheduled for convenience at specific times in the year),
- available to all customers.

Based on the definition, it must be concerned, that CMC assigned for a laboratory must reflect the capability of the respective laboratory in carrying out routine calibration to the nearly ideal measuring instrument or measurement standards, which can be calibrated by the laboratory using their own resources. Therefore, in practice, CMC is the uncertainty values, which often can be achieved by the laboratory in carrying out routine services.

Uncertainty reported by the laboratory may be smaller than their CMC, if in this case the laboratory calibrate measuring instruments or measurement standards, those have better characteristic than the nearly ideal condition used in the evaluation of CMC.

In certain condition, uncertainty reported by the laboratory may be larger than their CMC, if in this case laboratory calibrate measuring instruments or measurements standards those have worse characteristic than the nearly ideal condition used in the evaluation of CMC..

The cases, those need investigation seriously are when laboratory report much larger or much smaller uncertainty than their CMC for the calibration of measuring instruments or measurement standards those have equal or nearly equal characteristic with the nearly ideal condition used in the evaluation of CMC.

In practice CMC may be evaluated by measurement audit using nearly ideal artifacts or bay assessing uncertainty budget that usually used by the laboratory in carrying out routine services to their clients.

CMC consist of some components those depend on any factors needed by laboratory to demonstrate their competence. Those factors may include:

- Education, training and technical knowledge of personnel
- Environmental condition of calibration laboratory
- Maintenance of equipment, including calibration intervals and verifications

To get adequate evidence in assessing CMC, observation to the laboratory condition must be done by considering:

- Calibration Method
  Calibration method will affect CMC of the laboratory, because it usually states specification of unit under test, environmental condition requirements, calibrator, observation schemes, etc. The method used in the calibration processes will yield the different CMC values for the same reference standards or measuring equipment. For example, the CMC for the weight calibration based on the direct comparison method would be different from those based on the closed cycle or decade methods.

- Reference standard and measuring equipment Reference standards and measuring equipment used in the calibration processes are the major uncertainty sources in the evaluation of CMC. Their uncertainty will define the type of unit under test, which can be calibrated by the respective laboratory. In particular cases, laboratories those have same reference standard will have different CMC because of difference measuring equipment used. For example, mass calibration laboratories those have mass standards of E2 classes will have different CMC if a laboratory used mass comparator of 0.1 mg resolution and the other use 0.01 mg mass comparator.
  Beside the uncertainty stated in the calibration certificate, one important uncertainty source is drift of those reference standard and measuring equipment. It must be understood that the value stated in the certificates are only valid in the time of calibration. For the routine condition, the drift may occur, and it can be estimated based on the historical data.
- Ancillary equipment In the calibration processes, type and accuracy of ancillary equipment used to monitor influence quantities for the respective calibration will affect CMC values, as well as the data processing system for the data analysis. For example, in the weight calibration, ancillary equipment used to monitor the air density during calibration will give smaller CMC than the laboratory, which carry out weight calibration without air density monitoring system, and the uncertainty due to this factor estimated based on the worst condition of air density variation.
- Measurement techniques Different measurement techniques may cause the different CMC values, for example CMC for calibration of weight based on direct comparison method *Standard-Test-Test-Standard* carried once will give larger CMC than that carried out three series. If measurement carried out once, uncertainty due to repeatability will be (*stdev of balance* / $2^{1/2}$), and for three series of measurement will be (*stdev of balance* / $6^{1/2}$)
- Influence quantities Influence quantity is the quantity, which is not included in the definition of the measurand but affect the result of measurement. Thee quantities often cannot be eliminated perfectly so that the contribution must be taken into account in the uncertainty evaluation. For examples: for the calibration of mass standards based on the conventional weighing, the deviation of the laboratory condition from the air density of 1.2 kg/m^3 shall be taken into account.
- Personnel carry out calibration processes will contribute significant effect for the CMC evaluation. For example, different personnel in the weight calibration using the same mass standards and balance may get different result, because repeatability of balance obtained by two personnel’s may be different. In the

calibration of weights, the capability of personnel in observing the standard deviation balance will affect the routine calibration done by the laboratory.

- Specification of nearly ideal UUT, which can be calibrated by the laboratory The Definition of CMC stated that CMC assigned for the routine calibration of nearly ideal measurement standards or measuring instruments which can be calibrated by the respective laboratory. Based on the definition, contribution of the unit under test cannot be neglected in the CMC evaluation. For example, in the weight calibration, laboratory, which has mass standard of E2 class will has best capability to calibrate weight of F1 class, specification of mass standards give the specified densities range for each class of mass standards, in the CMC evaluation it mass be taken into account.

Different illustration may be given for the micrometer calibration using gauge block as reference standards based on JIS B 7502, in this case best condition of micrometer given by the standard must be taken into account.

Based on the observation results, the uncertainty sources for CMC evaluation may include, but not limited to:

1. Standard uncertainty due to the reference standard used in the respective calibration. These may include:
   - standard uncertainty of the calibration (based on the uncertainty reported in the calibration certificate)
   - *drift* of the reference standard (based on the historical data)
   - working condition of the reference standard
   -
2. Standard uncertainty due to the ancillary equipment, which have significant effect to the calibration results. These may include:
   - standard uncertainty of the calibration (based on the uncertainty reported in the calibration certificate)
   - *drift* of the ancillary equipment (based on the historical data)
   - working condition of the ancillary equipment
3. Type A standard uncertainty observed during the routine calibration processes in the respective laboratory, include the estimated type uncertainty of the nearly ideal unit under test.
4. Standard uncertainty due to the resolution, division or discrimination, include those come from the nearly ideal unit under test.
5. Standard uncertainty due to the other influence quantities and characteristics of the nearly ideal unit under test.

For the example of measurement uncertainty of Calibration :

- Weight see annex A1
- Electromechanical balance see annex A2
- Bourdon pressure see annex A3
- Outside micrometer see annex A4
- Thermocouple sensor see annex A5
- Digital multimeter see annex A6
- Vernier caliper see annex A7
- Platinum resistance thermometer see annex A8
- Volumetric glassware see annex A9

Testing :

- Weighing process see annex B1
- Volume preparation see annex B2
- Molecular weight see annex B3
- CG MS process see annex B4

18. REFERENCES

1. ISO *Guide to The Expression of Uncertainty in Measurement*, 1993, International Organization for Standardization, Geneva, Switzerland
2. ISO/IEC 17025 *General Requirements for the Competence of Testing and Calibration laboratories 2005*,
3. *ISO/IEC Guide 99 2007*
4. Taylor, B N, Kuyatt, C E, *Guideline for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, NIST Technical Note 1297, 1993
5. SAC-SINGLAS Technical Guide 1, *Guidelines of The Evaluation and Expression of Measurement Uncertainty*, 2nd edition, 2001
6. SAC-SINGLAS Technical Guide 2, *Guidelines of The Evaluation and Expression of Uncertainty in Chemical Analysis*, 1st edition, 2000
7. EA-4/02 *Expression of The Uncertainty of Measurement in Calibration*, European Accreditation, 1999
8. Cook, R R, *Assessment of Uncertainty of Measurement for Calibration and Testing Laboratories*, 1998
9. EURACHEM/CITAC Guide *Quantifying Uncertainty in Analytical Measurement*, 2000
10. Cook, R R, Giardini, W J, *Guide to the ISO Guide to the Expression of Uncertainty in Measurement*, CSIRO-NML 1993